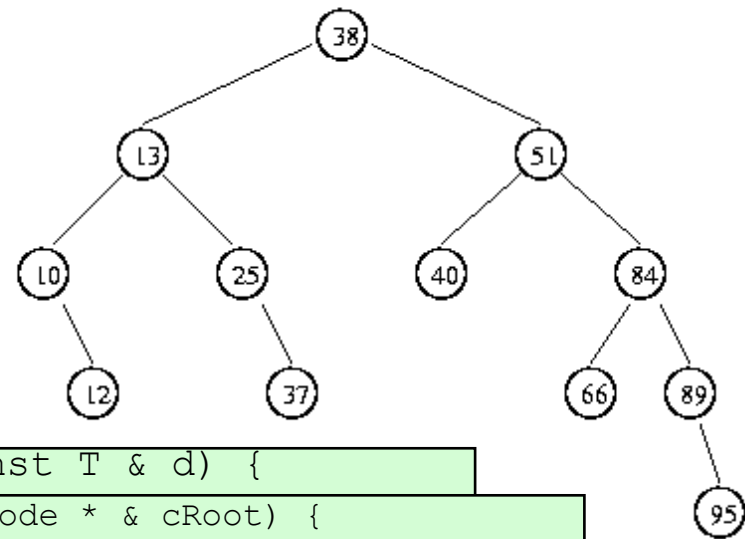


Announcements

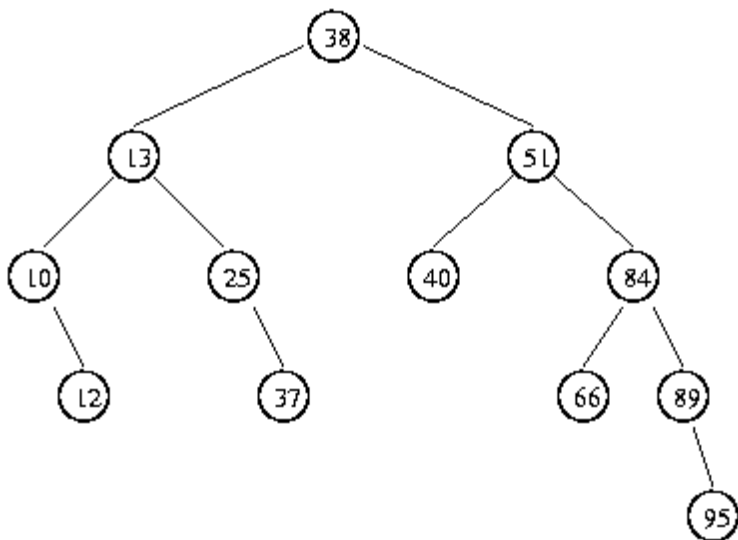
MP5 available, due 11/1, 11:59p. EC due 10/25, 11:59p.

<http://www.qmatica.com/DataStructures/Trees/AVL/AVLTree.html>



```
void Dict<K,V>::remove(treeNode * & cRoot, const T & d) {  
    if (cRoot != NULL) {  
        if (cRoot->key == d) {  
            doRemoval(cRoot);  
        }  
        else if (d < cRoot->key) {  
            remove(cRoot->left, d);  
        }  
        else {  
            remove(cRoot->right, d);  
        }  
    }  
}  
  
void Dictionary<K>::doRemoval(treeNode * & cRoot) {  
    if ((cRoot->left == NULL) && (cRoot->right == NULL)) {  
        delete cRoot;  
        cRoot = NULL;  
    }  
    else if (cRoot->left != NULL && cRoot->right == NULL) {  
        twoChildRemove(cRoot->left, cRoot);  
    }  
    else if (cRoot->left == NULL && cRoot->right != NULL) {  
        twoChildRemove(cRoot->right, cRoot);  
    }  
    else {  
        oneChildRemove(cRoot);  
    }  
}  
  
void Dict<K,V>::twoChildRemove(treeNode * & cRoot) {  
    treeNode * iop = rightMostChild(cRoot->left);  
    cRoot->key = iop->key;  
    doRemoval(iop);  
}
```

Binary Search Tree - Remove



```
void Dict<K,V>::remove(treeNode * & cRoot, const T & d) {
```

```
    if (cRoot != NULL) {
```

```
        if (cRoot->key == d) {
```

```
            doRemoval(cRoot);
```

```
        } else if (cRoot->key < d) {
```

```
            remove(d, cRoot->right);
```

```
        } else {
```

```
            remove(d, cRoot->left);
```

```
    }
```

```
void Dictionary<K>::doRemoval(treeNode * & cRoot) {
```

```
    if ((cRoot->left == NULL) && (cRoot->right == NULL)) {
```

```
        delete cRoot;
```

```
    } else if (cRoot->key == d) {
```

```
        twoChildRemove(cRoot);
```

```
    } else {
```

```
        oneChildRemove(cRoot, d);
```

```
void Dict<K,V>::twoChildRemove(treeNode * & cRoot) {
```

```
    treeNode * iop = rightMostChild(cRoot->left);
```

```
    cRoot->key = iop->key;
```

```
    doRemoval(iop);
```

```
}
```

```
treeNode * & Dict<K,V>::rightMostChild(treeNode * & cRoot) {
```

```
    if (cRoot->right == NULL) return cRoot;
```

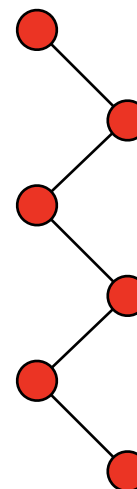
```
    else return rightMostChild(cRoot->right);
```

```
}
```

Binary Search Tree - miscellaneous characteristics and analysis

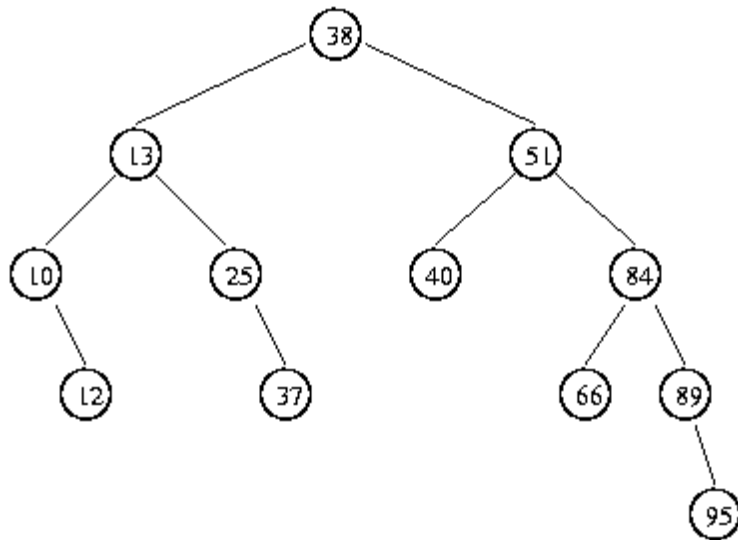
```
BST<int> myT;  
myT.insert(2);  
myT.insert(7);  
myT.insert(15);  
myT.insert(22);  
myT.insert(28);  
...
```

Give a sequence of inserts that result in a tree that looks like:



How many “bad” n-item trees are there?

Binary Tree -



The *algorithms* on BST depend on the height (h) of the tree.

The *analysis* should be in terms of the amt of data (n) the tree contains.

So we need relationships between h and n.

$$h \geq f(n)$$

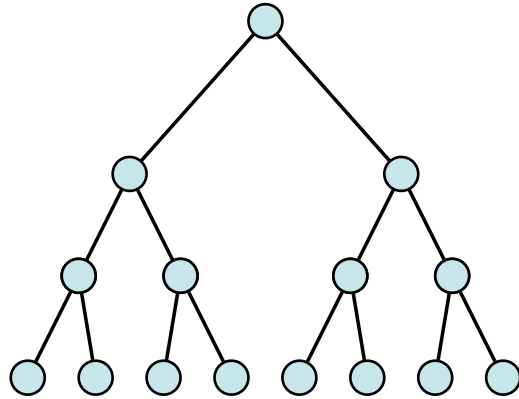
$$h \leq g(n)$$

Reminder:

height(T) is:

- _____ if T is empty
- $1 + \max\{\text{height}(T_L), \text{height}(T_R)\}$, otherwise

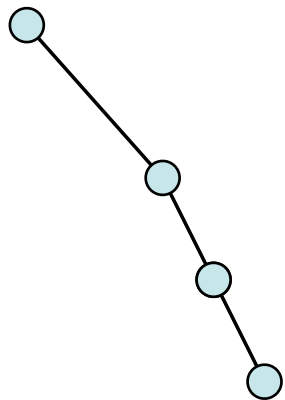
Binary Tree (theory moment #1)



what is maximum number of nodes in a tree of height h ?

what is the least possible height (h) for a tree of n nodes?

Binary Tree (theory moment #2)



what is minimum number of nodes (n) in a tree of height h ?

what is the greatest possible height (h) for a tree of n nodes?

thus: lower bd on ht _____, upper bd on ht _____, good news or bad?

Binary Search Tree -

The height of a BST depends on the order in which the data is inserted into it.

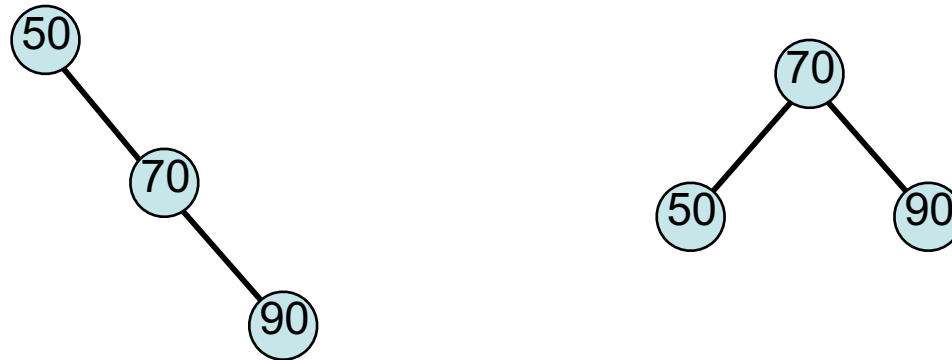
ex. 1 3 2 4 5 7 6 vs. 4 2 3 6 7 1 5

How many different ways are there to insert n keys into a tree?

Avg height, over all arrangements of n keys is _____.

operation	avg case	worst case	sorted array	sorted list
find				
insert				
delete				
traverse				

something new... which tree makes you happiest?



The “height balance” of a tree T is:

$$b = \text{height}(T_L) - \text{height}(T_R)$$

A tree T is “height balanced” if:

-
-