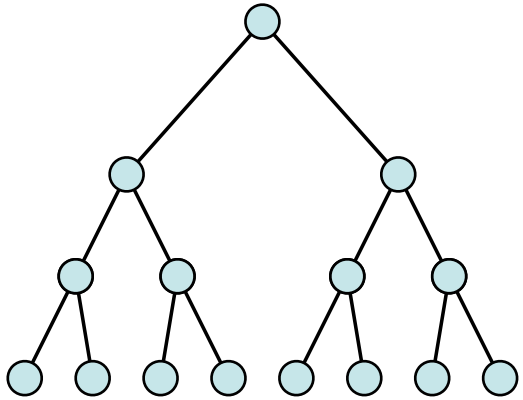
# Announcements

MP5 available, due 11/1, 11:59p. EC due 10/25, 11:59p.

http://www.qmatica.com/DataStructures/Trees/AVL/AVLTree.html

Binary Tree (theory moment) we want to find f(n) so that h ≥ f(n).



what is maximum number of nodes in a tree of height h? (a seemingly unrelated question)

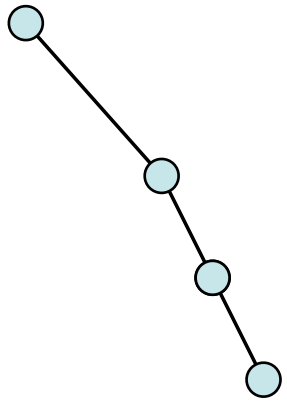
M(h): max number of nodes in tree of ht h, h ≥-1

M(-1) = 0, M(h) = 1 + 2M(h-1), h > -1.

Hypothesize $M(h) = 2^{(h+1)} - 1$, h ≥-1.

what is the least possible height (h) for a tree of n nodes?

# Binary Tree (theory moment #2)



what is minimum number of nodes (n) in a tree of height h?

what is the greatest possible height (h) for a tree of n nodes?

thus:  lower bd on ht _____, upper bd on ht _____,  good news or bad?

# Binary Search Tree -

The height of a BST depends on the order in which the data is inserted into it.

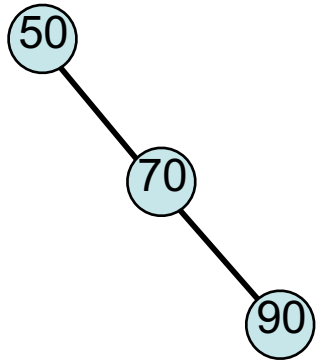
ex. 1 3 2 4 5 7 6  vs.  4 2 3 6 7 1 5

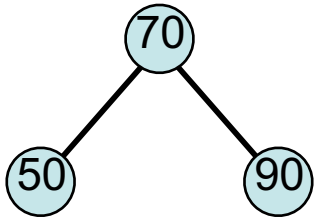
How many different ways are there to insert n keys into a tree?

Avg height, over all arrangements of n keys is _____.

| operation | avg case | worst case | sorted array | sorted list |
|---|---|---|---|---|
| find | | | | |
| insert | | | | |
| delete | | | | |
| traverse | | | | |

something new… which tree makes you happiest?



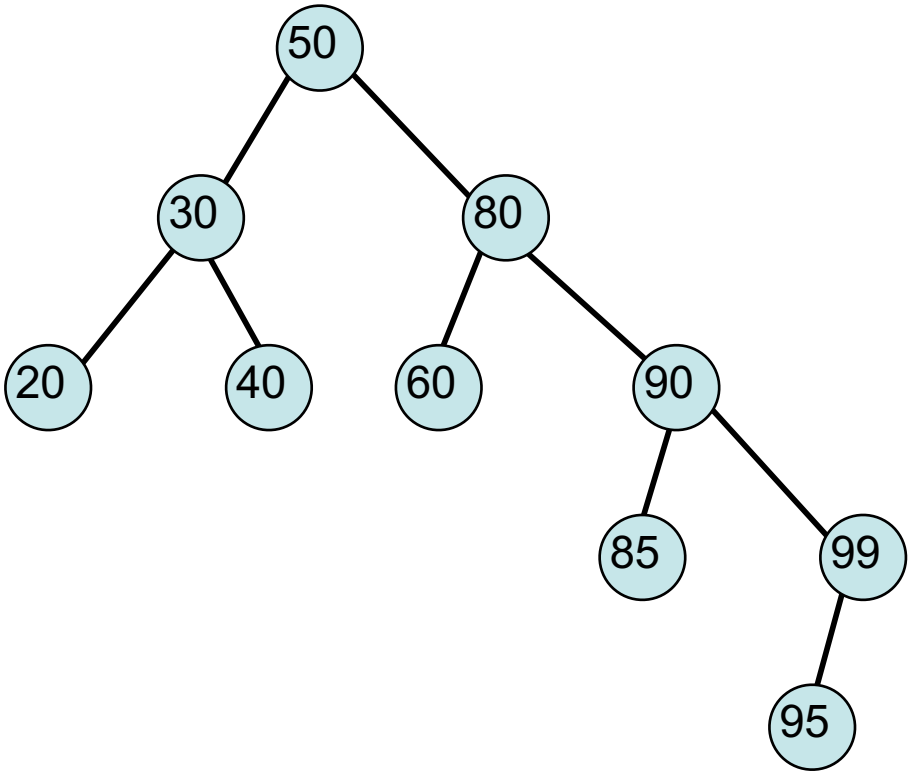

The “height balance” of a tree T is:

$$b = height(T_R) - height(T_L)$$

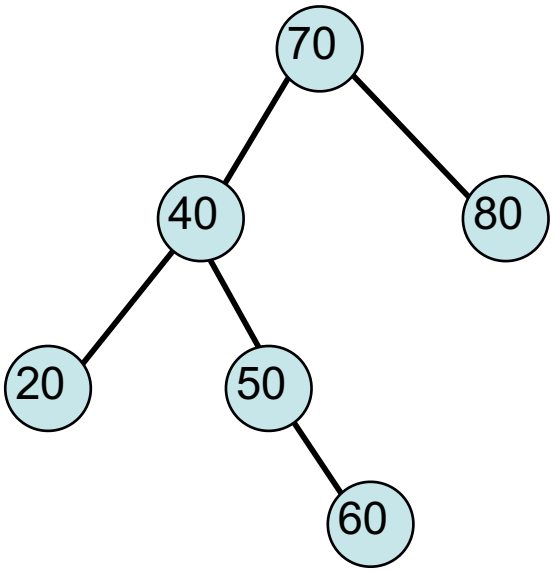
A tree T is “height balanced” if:

- T = {} OR
- T = {r, $T_L$, $T_R$}, _____, and $T_L$ and $T_R$ are ht balanced.

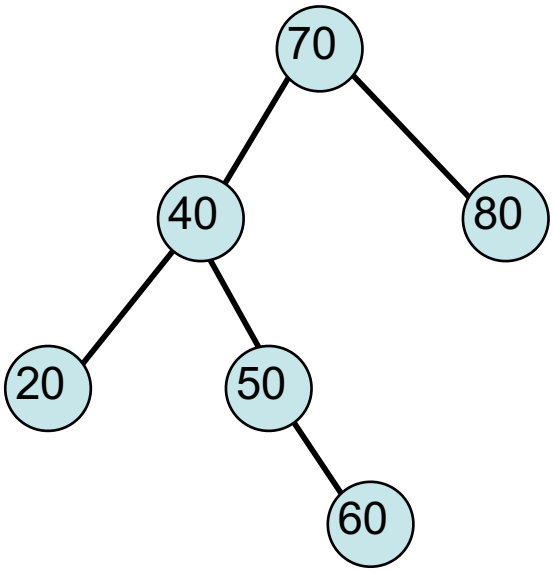
# operations on BST - rotations



# balanced trees - rotations



# balanced trees - rotations



balanced trees - rotations summary:

- there are 4 kinds: left, right, left-right, right-left (symmetric!)
- local operations (subtrees not affected)
- constant time operations
- BST characteristic maintained

GOAL: use rotations to maintain balance of BSTs.

height balanced trees - we have a special name:

Three issues to consider as we move toward implementation:

Rotating

Maintaining height

Detecting imbalance