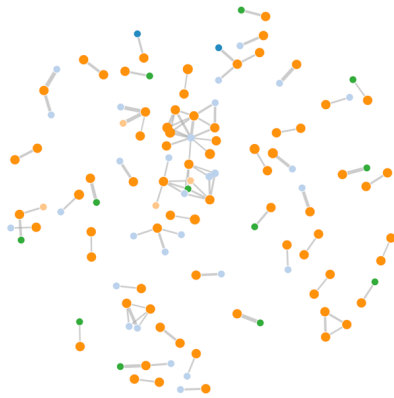
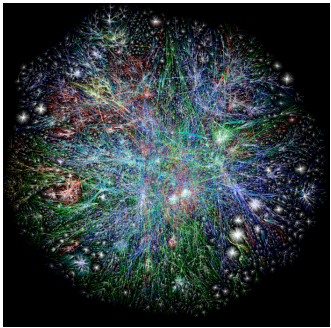


Today's announcements:

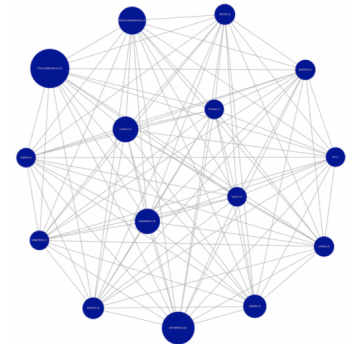
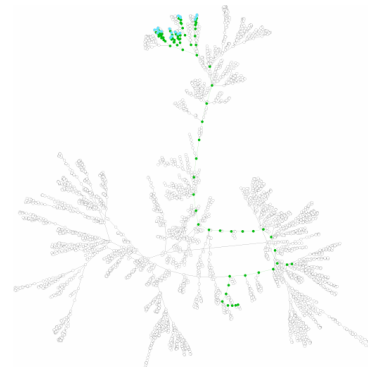
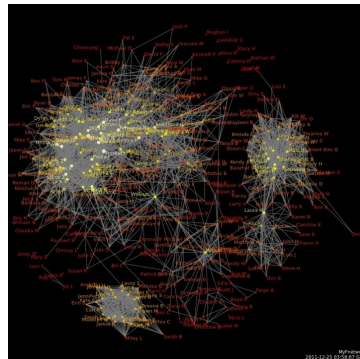
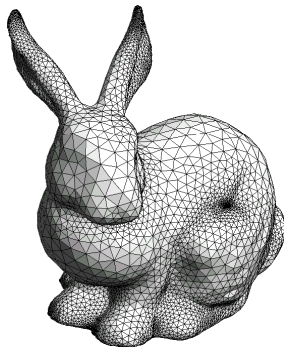
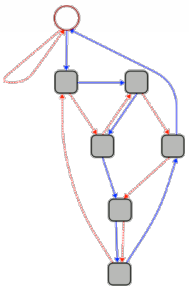
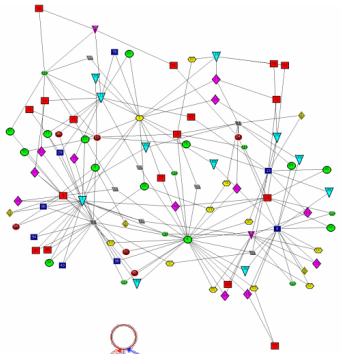
MP7 available. Due 12/6, 11:59p. EC due 11/30, 11:59p.

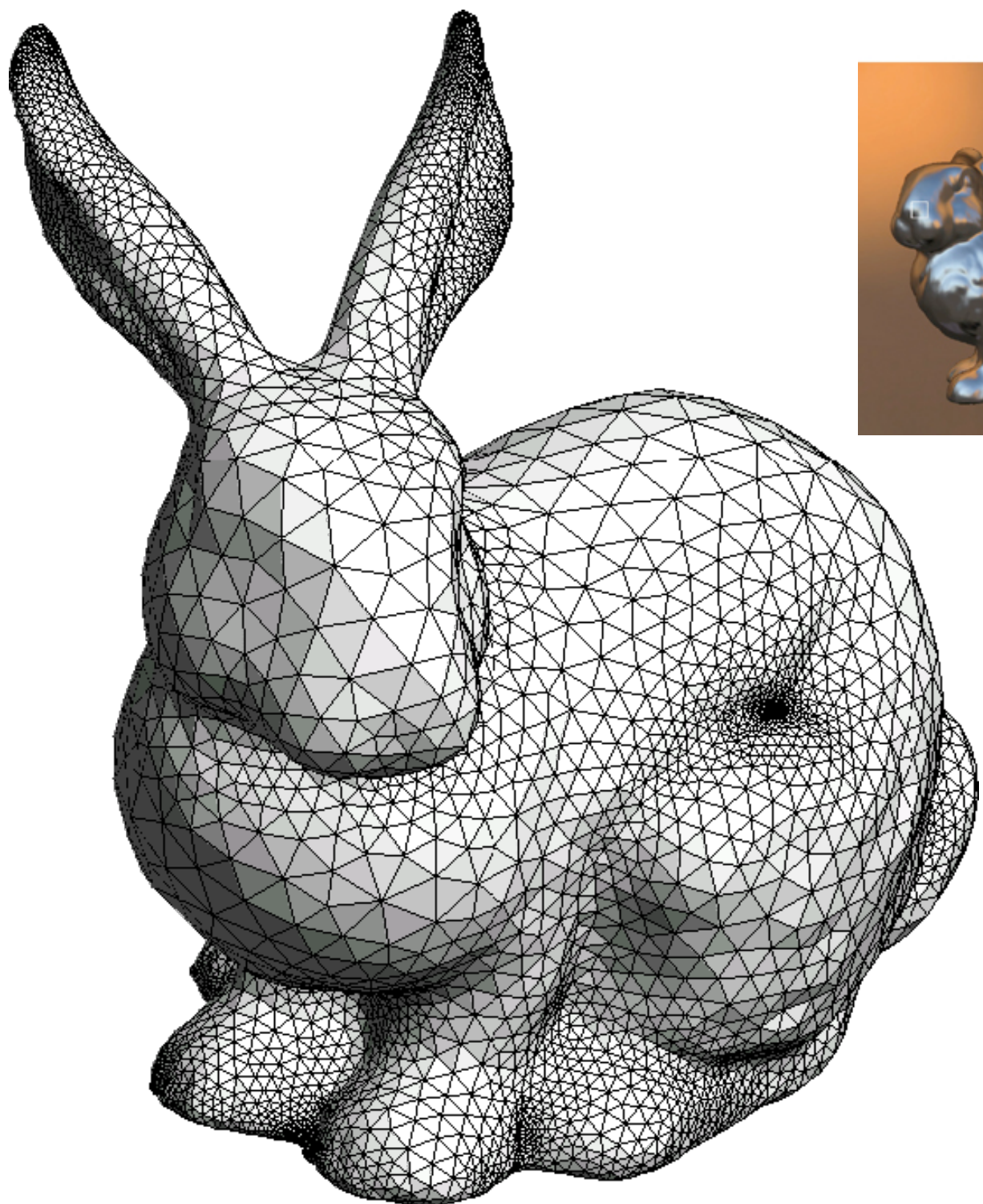
Exam5: 12/4-12/7, CBTF.



How do we get from here to there?
Need:

1. *Common Vocabulary*
2. *Graph implementation*
3. *Traversal*
4. *Algorithms.*





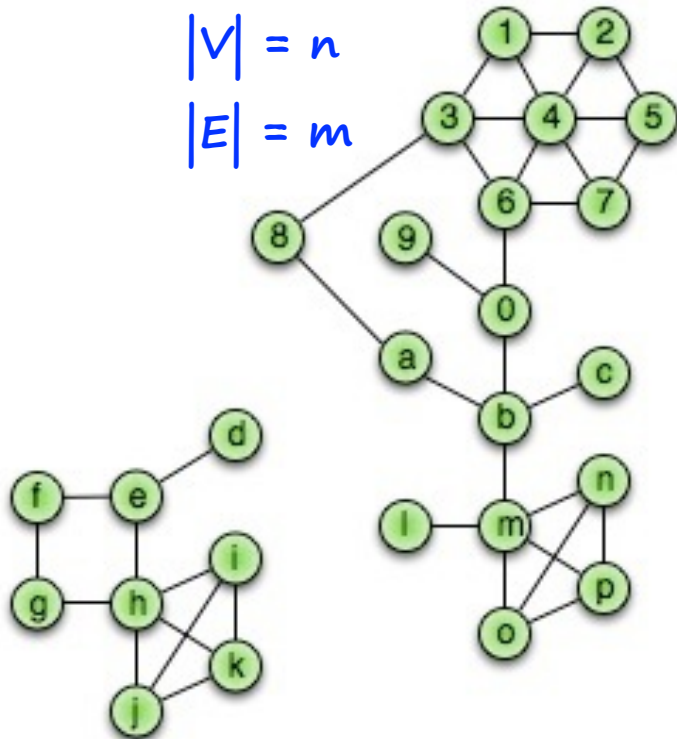
Incident edges(v): $I = \{(x,v) \text{ in } E\}$

Graph Vocabulary:

$$G = (V, E)$$

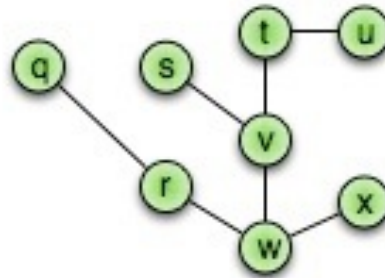
$$|V| = n$$

$$|E| = m$$



G_1

G_2



G_3

Degree(v): $|I|$

Adjacent vertices(v): $A = \{x: (x,v) \text{ in } E\}$

Path(G_2) - sequence of vertices connected by edges.

Cycle(G_1) - path with common begin and end vertex.

Simple graph(G) - graph with no self-loops and no multi-edges.

Graph Vocabulary:

Subgraph(G) – $G' = (V', E')$, $V' \subseteq V$, $E' \subseteq E$, and $(u,v) \in E'$ implies $u \in V'$ and $v \in V'$.

Complete subgraph(G_2) –

Connected subgraph(G) –

Connected component(G) –

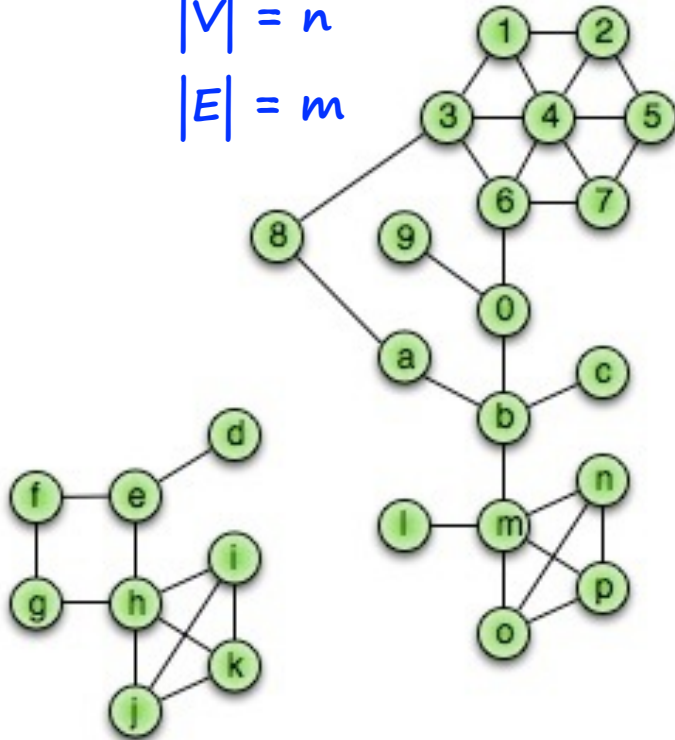
Acyclic subgraph(G_2) –

Spanning tree(G_1) –

$$G = (V, E)$$

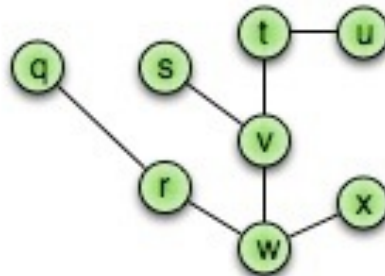
$$|V| = n$$

$$|E| = m$$



G_1

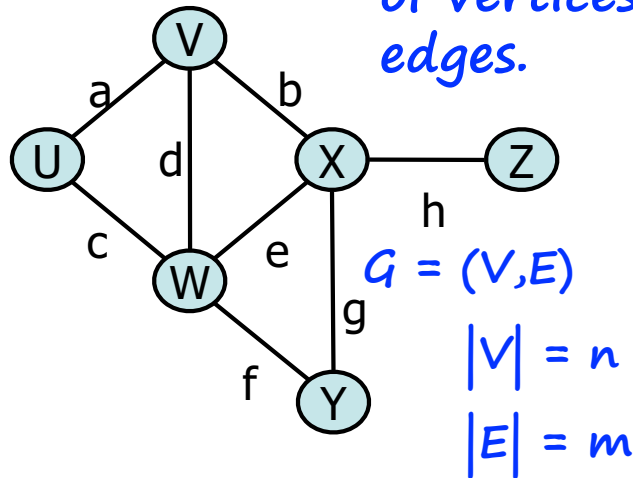
G_2



G_3

Graphs: theory that will help us in analysis

Running times often reported in terms of n , the number of vertices, but they often depend on m , the number of edges.



How many edges?

At least:

connected –

not connected -

At most:

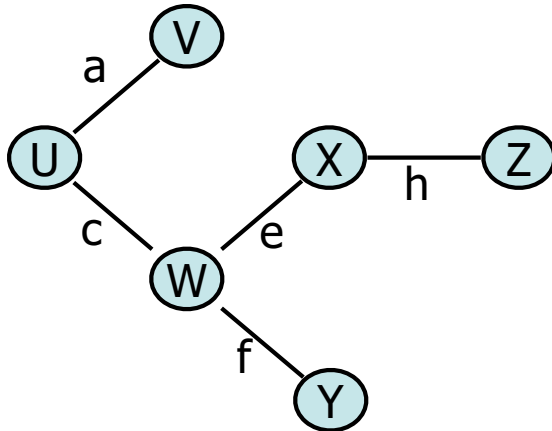
simple -

not simple -

Relationship to degree sum:

$$\sum_{v \in V} \deg(v) =$$

Thm: Every minimally connected graph $G=(V,E)$ has $|V|-1$ edges.



Proof: Consider an arbitrary minimally connected graph $G=(V,E)$.

Lemma: Every connected subgraph of G is minimally connected. (easy proof by contradiction)

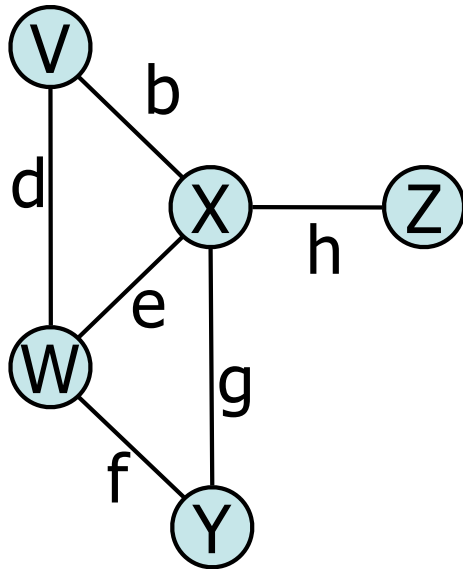
IH: For any $j < |V|$, any minimally connected graph of j vertices has $j-1$ edges.

Suppose $|V| = 1$: A minimally connected graph of 1 vertex has no edges, and $0 = 1-1$.

Suppose $|V| > 1$: Choose any vertex and let d denote its degree. Remove its incident edges, partitioning the graph into _____ components, $C_0=(\text{____}, \text{____})$, ... $C_d=(\text{____}, \text{____})$, each of which is a minimally connected subgraph of G . This means that $|E_k| = \text{_____}$ by _____.

Now we'll just add up edges in the original graph:

Graphs: Toward implementation...(ADT)



Data:

Vertices

Edges

+ some structure that reflects
the connectivity of the graph

Functions: (merely a smattering...)

`insertVertex(pair keyData)`

`insertEdge(vertex v1, vertex v2, pair keyData)`

`removeEdge(edge e);`

`removeVertex(vertex v);`

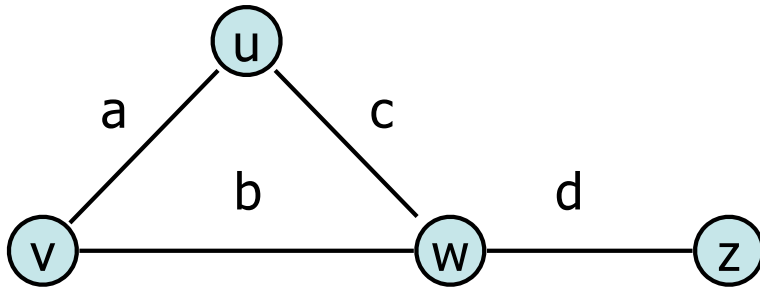
`incidentEdges(vertex v);`

`areAdjacent(vertex v1, vertex v2);`

`origin(edge e);`

`destination(edge e);`

Graphs: Edge List (a first implementation)



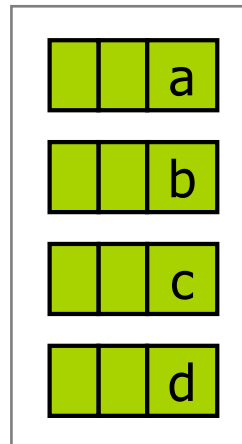
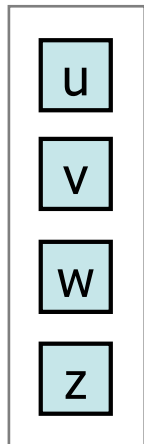
Some functions we'll compare:

`insertVertex(vertex v)`

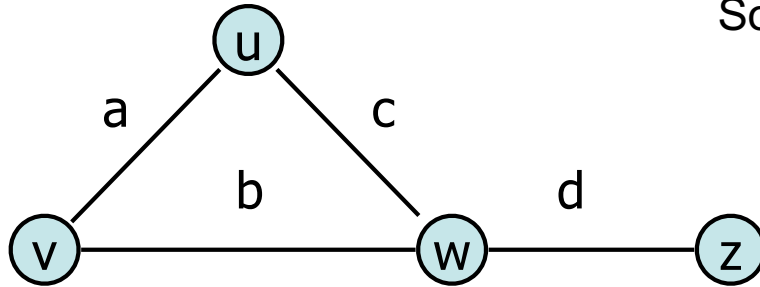
`removeVertex(vertex v)`

`areAdjacent(vertex v, vertex u)`

`incidentEdges(vertex v)`



Graphs: Adjacency Matrix



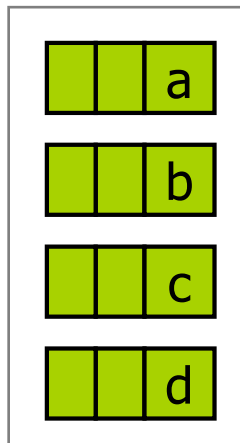
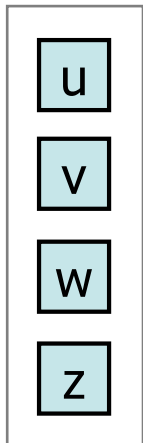
Some functions we'll compare:

`insertVertex(vertex v)`

`removeVertex(vertex v)`

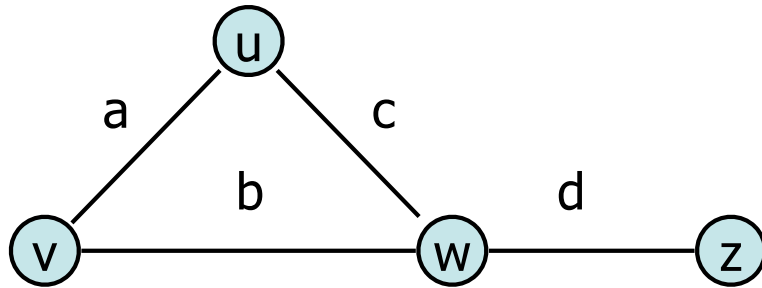
`areAdjacent(vertex v, vertex u)`

`incidentEdges(vertex v)`



	u	v	w	z
u				
v				
w				
z				

Graphs: Adjacency List



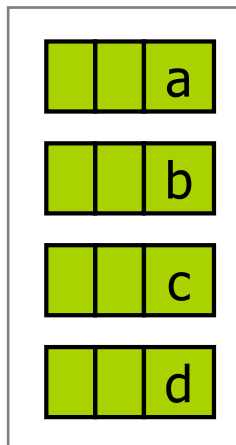
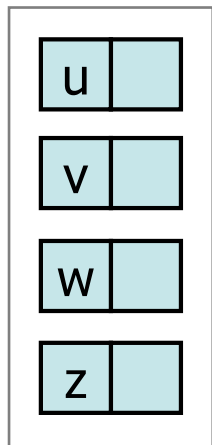
Some functions we'll compare:

`insertVertex(vertex v)`

`removeVertex(vertex v)`

`areAdjacent(vertex v, vertex u)`

`incidentEdges(vertex v)`



Graphs: Asymptotic Performance

• n vertices, m edges • no parallel edges • no self-loops • Bounds are big-O	Edge List	Adjacency List	Adjacency Matrix
Space	$n + m$	$n + m$	n^2
incidentEdges(v)	m	$\deg(v)$	n
areAdjacent (v, w)	m	$\min(\deg(v), \deg(w))$	1
insertVertex(o)	1	1	n^2
insertEdge(v, w, o)	1	1	1
removeVertex(v)	m	$\deg(v)$	n^2
removeEdge(e)	1	1	1