Today's announcements:

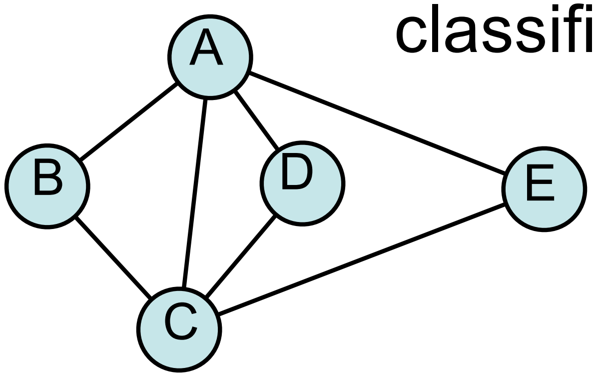
MP7 available. Due 12/6, 11:59p.

Exam5: 12/4-12/7, CBTF. Review Fri, 12/2, 7-9p, Siebel 1404

Final: 12/8on, CBTF. Review Sat, 12/3, 4-6p, Loomis 141

Graphs: Traversal - DFS

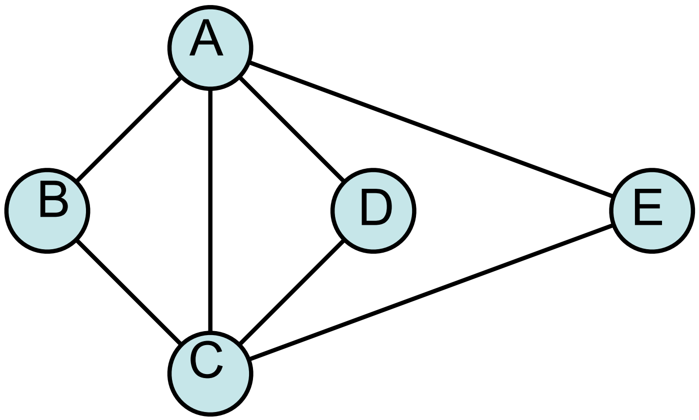
DFS: "visits" each vertex
classifies each edge as either "discovery" or "back"




```
Algorithm DFS(G)
        Input: graph G
        Output:  labeling of the edges of G
as discovery edges and back edges


For all u in G.vertices()
  setLabel(u, UNVISITED)
For all e in G.edges()
  setLabel(e, UNEXPLORED)
For all v in G.vertices()
  if getLabel(v) = UNVISITED
    DFS(G,v)
```

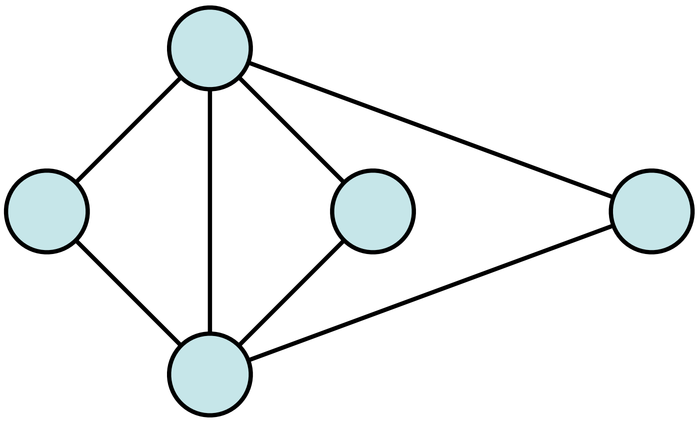
```
Algorithm DFS(G,v)
        Input: graph G and start vertex v
        Output:  labeling of the edges of G in the
connected component of v as discovery edges and
back edges


setLabel(v, VISITED)
For all w in G.adjacentVertices(v)
    if getLabel(w) = UNVISITED
      setLabel((v,w),DISCOVERY)
      DFS(G,w)
    else if getLabel((v,w)) = UNEXPLORED
      setLabel(e,BACK)
```

Graphs: DFS example



A	B C D E
B	A C
C	B A D E
D	A C
E	A C

Graphs: DFS Analysis



setting/getting labels

- every vertex labeled twice

- every edge is labeled twice

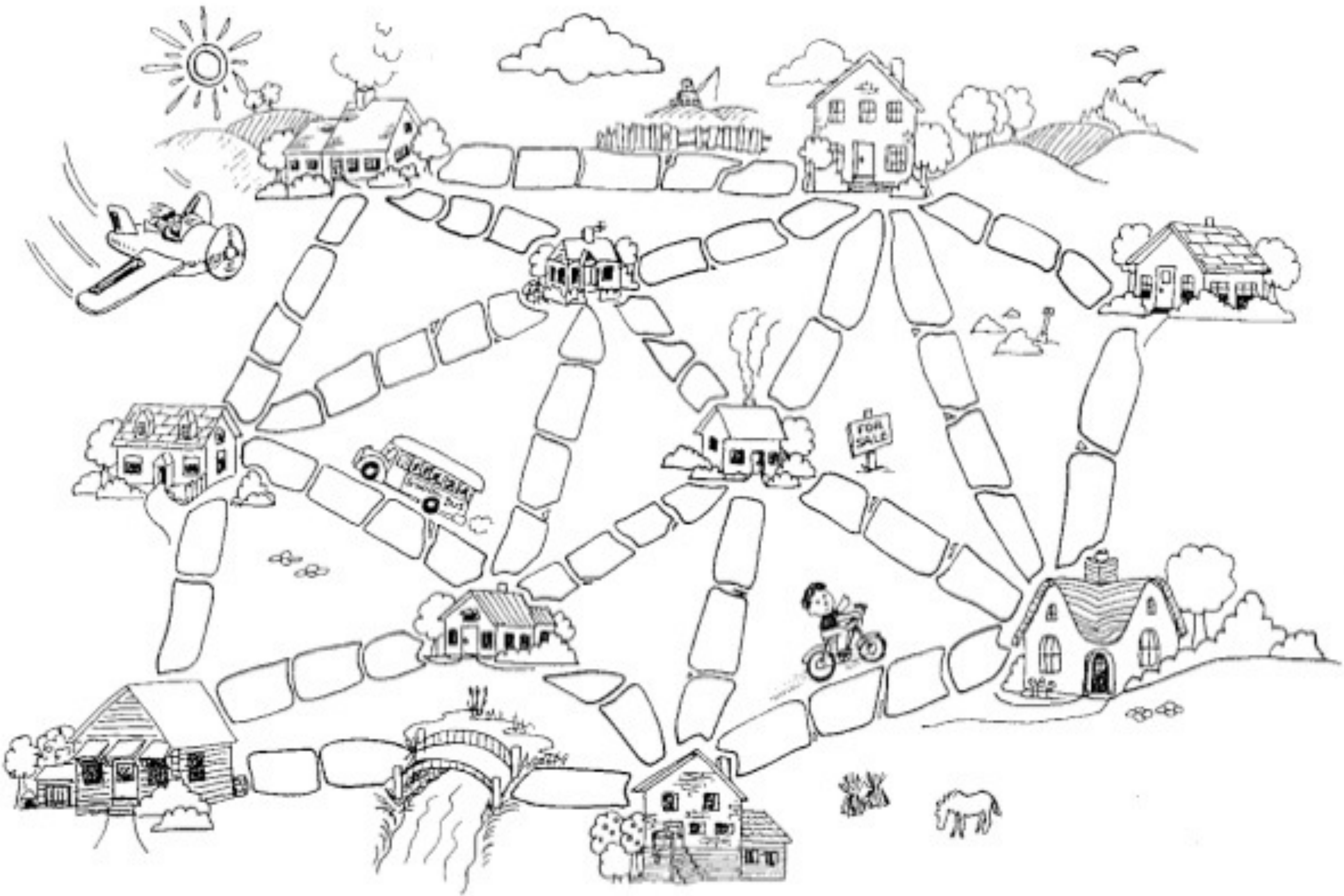
querying vertices

- each vertex
- total over algorithm

querying edges

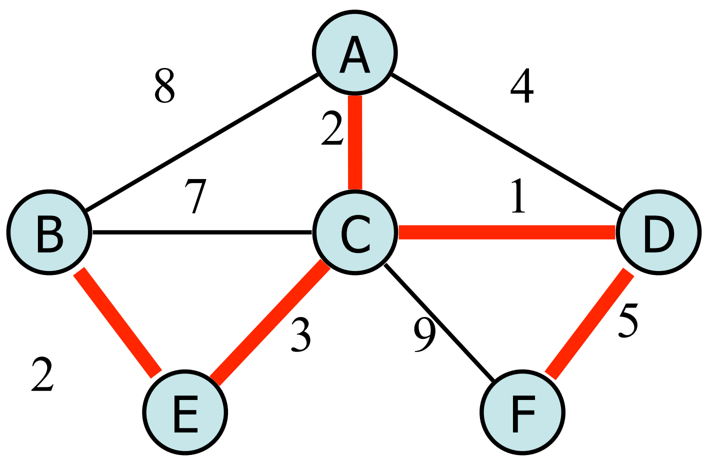
TOTAL RUNNING TIME:

Muddy City…

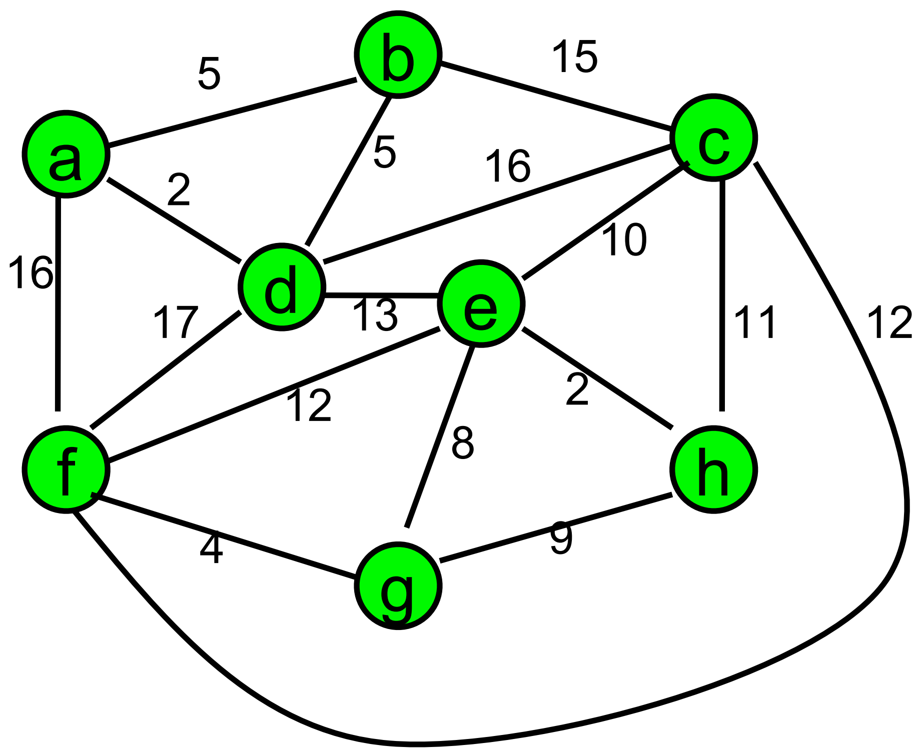


Minimum Spanning Tree Algorithms:

- Input: connected, undirected graph G with unconstrained edge weights
- Output: a graph G' with the following characteristics -
  - G' is a spanning subgraph of G
  - G' is connected and acyclic (a tree)
  - G' has minimal total weight among all such spanning trees -

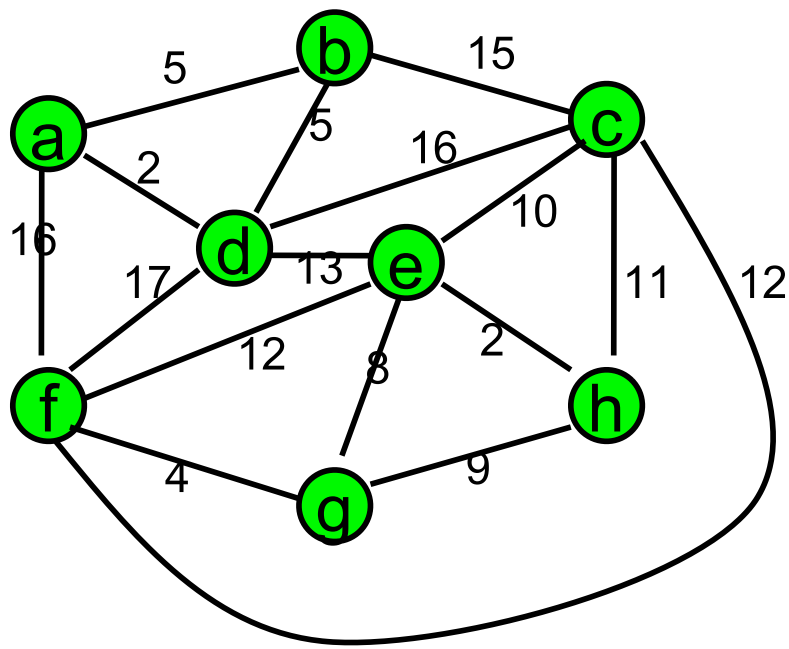


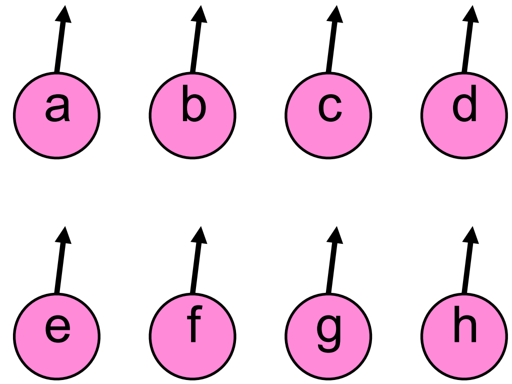
Kruskal’s Algorithm



(a,d)
(e,h)
(f,g)
(a,b)
(b,d)
(g,e)
(g,h)
(e,c)
(c,h)
(e,f)
(f,c)
(d,e)
(b,c)
(c,d)
(a,f)
(d,f)

Kruskal's Algorithm (1956)





(a,d)
(e,h)
(f,g)
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(g,e)
(g,h)
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(c,h)
(e,f)
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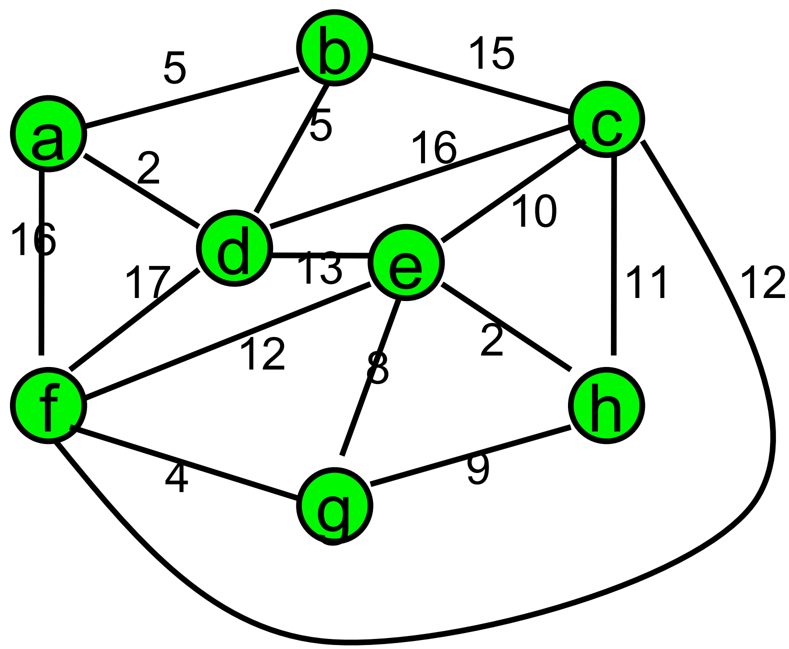
(d,e)
(b,c)
(c,d)
(a,f)
(d,f)

1. Initialize graph T whose purpose is to be our output. Let it consist of all n vertices and no edges.

2. Initialize a disjoint sets structure where each vertex is represented by a set.

3. RemoveMin from PQ. If that edge connects 2 vertices from different sets, add the edge to T and take Union of the vertices' two sets, otherwise do nothing. Repeat until _____ edges are added to T.

Kruskal's Algorithm - preanalysis



Algorithm *KruskalMST*(*G*)

disjointSets forest;
for each vertex ***v*** in ***V*** **do**
  forest.makeSet(v);

priorityQueue Q;
Insert edges into ***Q***, keyed by weights

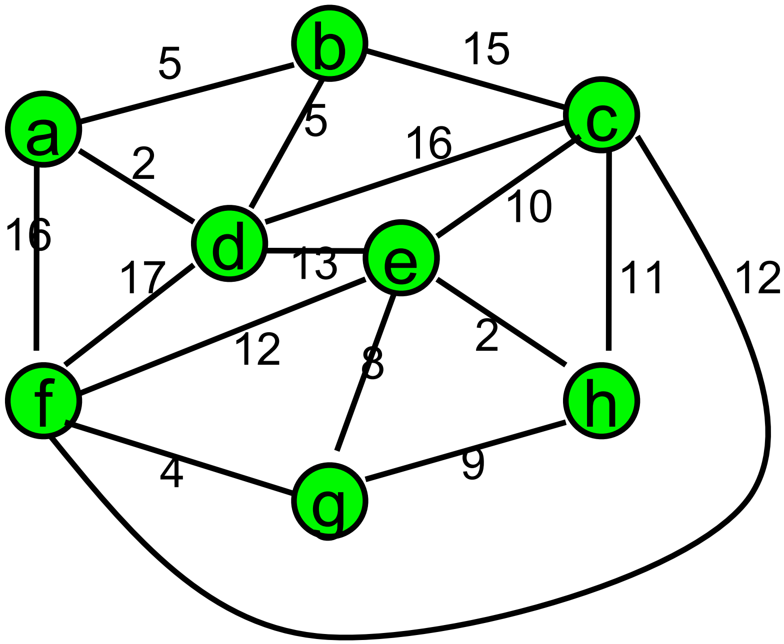
graph ***T = (V,E)*** with ***E*** = ∅;

while ***T*** has fewer than ***n***-1 edges **do**
  edge ***e*** = ***Q.removeMin()***
  Let ***u***, ***v*** be the endpoints of *e*
  if ***forest.find(v) ≠ forest.find(u)*** **then**
    Add edge ***e*** to ***E***
    forest.smartUnion
      (forest.find(v),forest.find(u))

return ***T***

Priority Queue:	Heap	Sorted Array
To build		
Each removeMin		

Kruskal's Algorithm - analysis



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    forest.smartUnion
      (forest.find(v),forest.find(u))

return ***T***

Priority Queue:	Total Running time:
Heap	
Sorted Array	