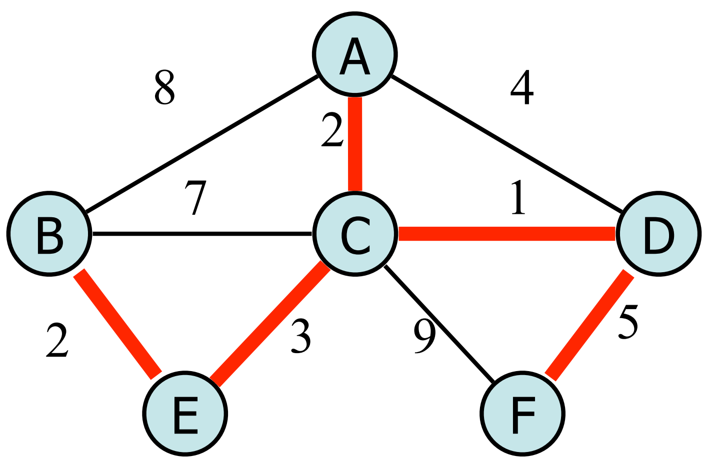
Today's announcements:

MP7 available. Due 12/6, 11:59p.

Exam5: 12/4-12/7, CBTF. Final: 12/8on, CBTF.

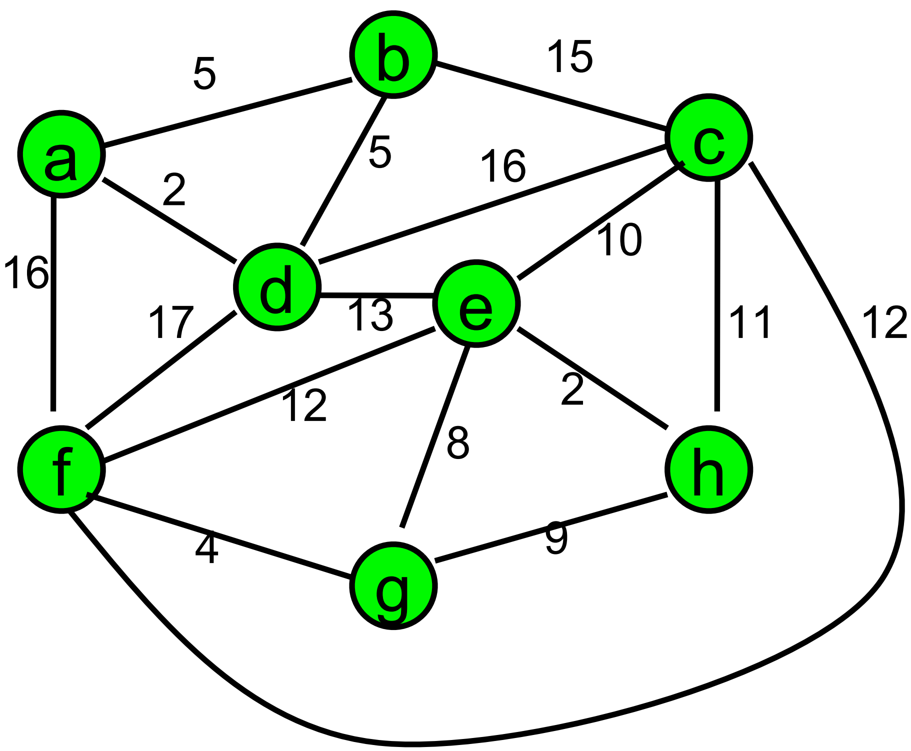
Minimum Spanning Tree Algorithms:

- Input: connected, undirected graph G with unconstrained edge weights
- Output: a graph G' with the following characteristics -
  - G' is a spanning subgraph of G
  - G' is connected and acyclic (a tree)
  - G' has minimal total weight among all such spanning trees -



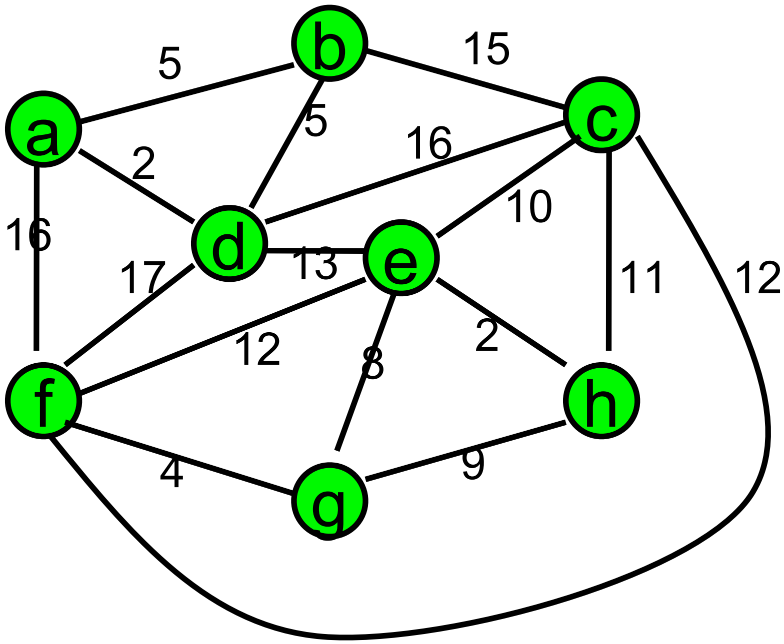
____

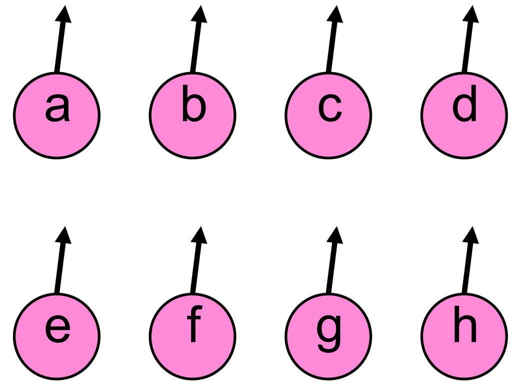
Kruskal's Algorithm



(a,d)
(e,h)
(f,g)
(a,b)
(b,d)
(g,e)
(g,h)
(e,c)
(c,h)
(e,f)
(f,c)
(d,e)
(b,c)
(c,d)
(a,f)
(d,f)

Kruskal's Algorithm (1956)





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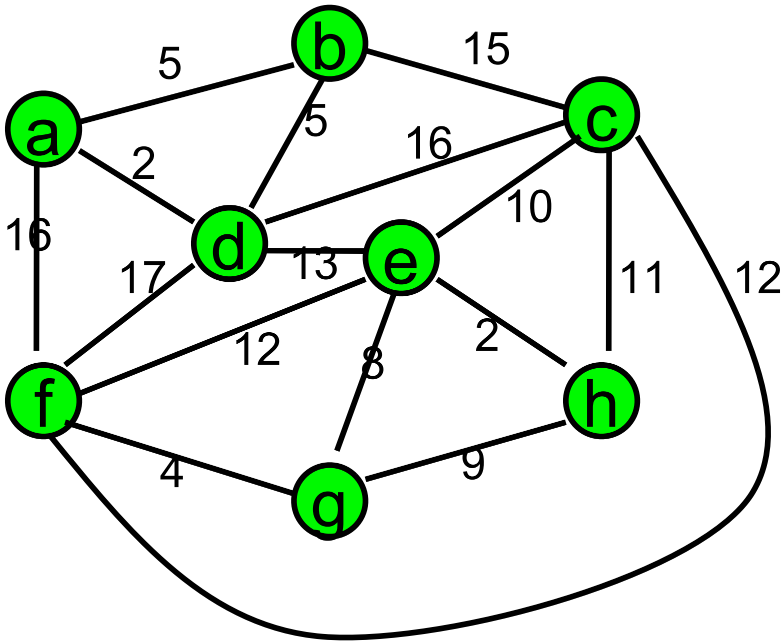
(b,c)
(c,d)
(a,f)
(d,f)

1. Initialize graph T whose purpose is to be our output. Let it consist of all n vertices and no edges.

2. Initialize a disjoint sets structure where each vertex is represented by a set.

3. RemoveMin from PQ. If that edge connects 2 vertices from different sets, add the edge to T and take Union of the vertices' two sets, otherwise do nothing. Repeat until _____ edges are added to T.

Kruskal's Algorithm - preanalysis



Algorithm ***KruskalMST***(***G***)

disjointSets forest;
for each vertex ***v*** in ***V*** **do**
  forest.makeSet(v);

priorityQueue Q;
Insert edges into ***Q***, keyed by weights

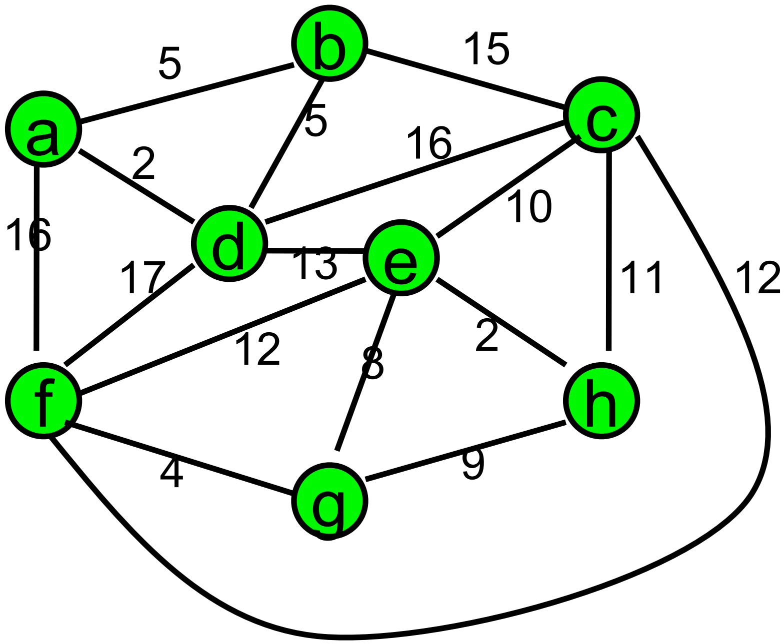
graph T = (V,E) with ***E*** = ∅;

while ***T*** has fewer than ***n***-1 edges **do**
  edge ***e*** = ***Q.removeMin()***
  Let ***u***, ***v*** be the endpoints of *e*
  if *forest.find(v) ≠ forest.find(u)* **then**
    Add edge ***e*** to ***E***
    forest.smartUnion
      (forest.find(v),forest.find(u))

return ***T***

Priority Queue:	Heap	Sorted Array
To build		
Each removeMin		

Kruskal's Algorithm - analysis



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return ***T***

Priority Queue:	Total Running time:
Heap	
Sorted Array	

Prim's algorithms (1957) is based on the Partition Property:

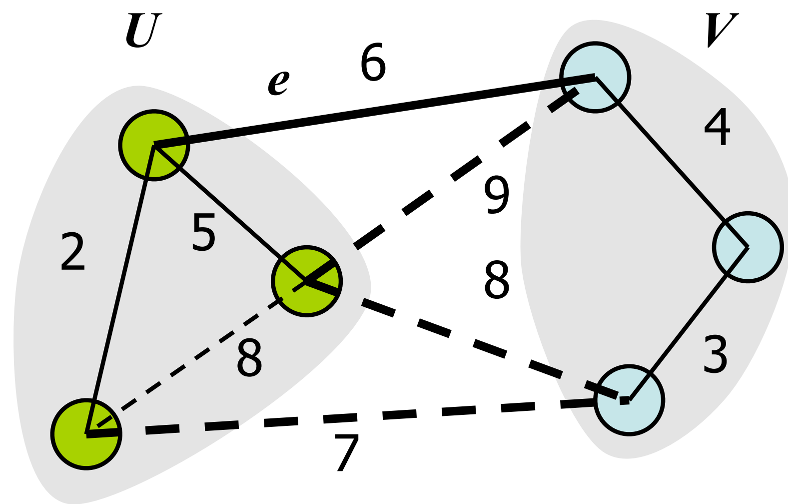
Consider a partition of the vertices of G into subsets U and V.

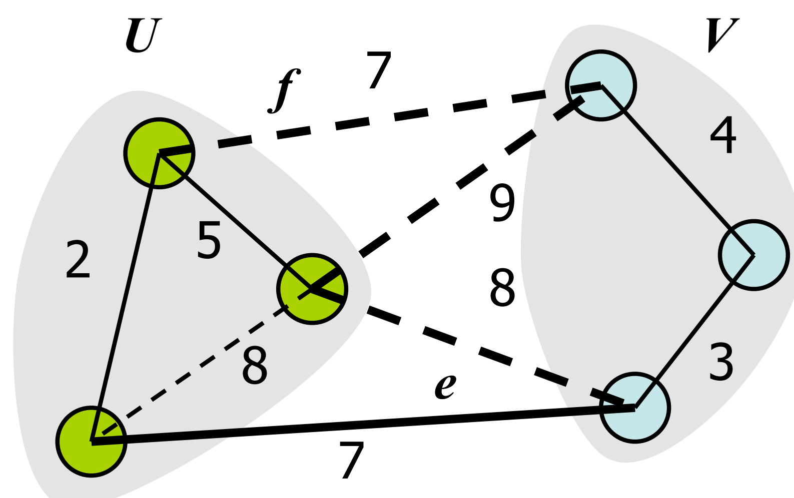
Let e be an edge of minimum weight across the partition.

Then e is part of some minimum spanning tree.

Proof:

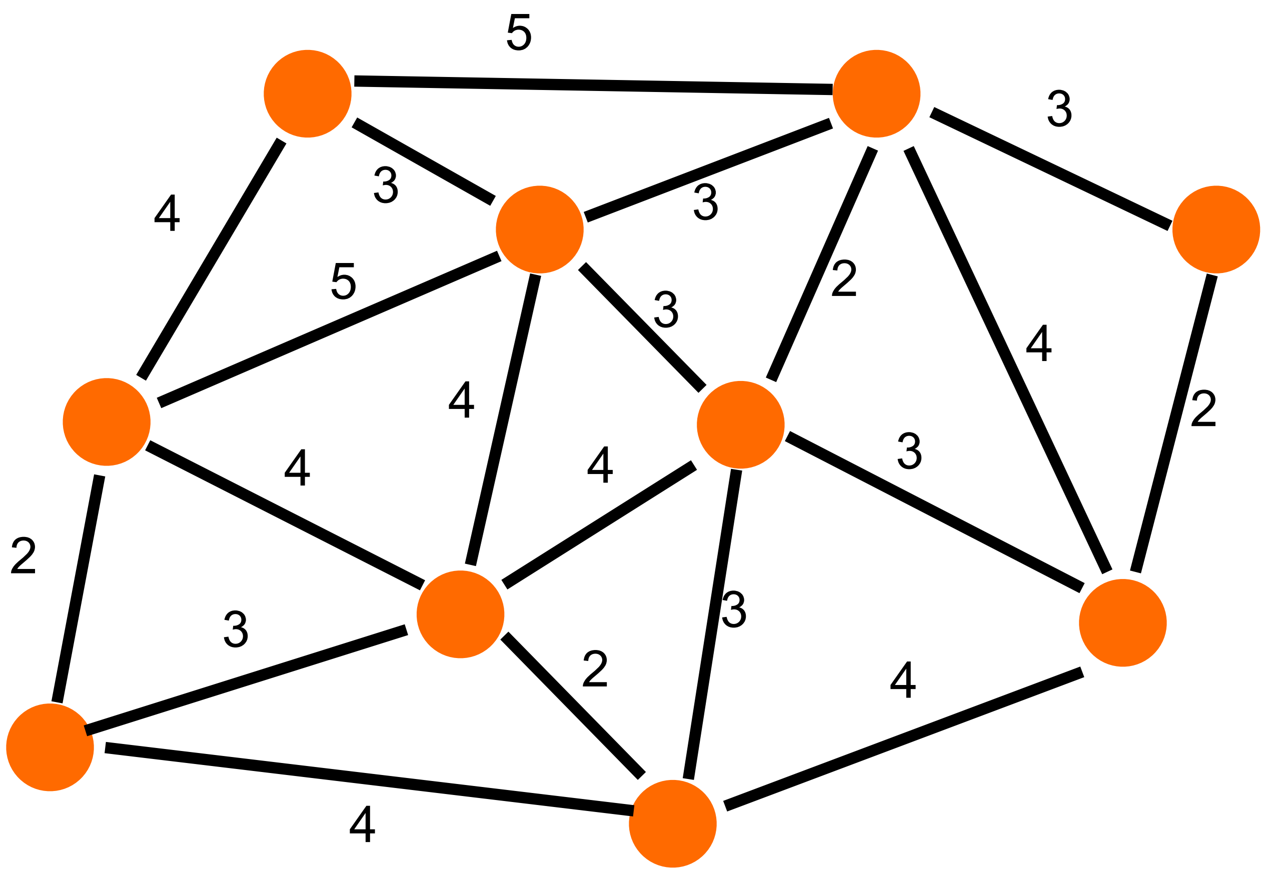
See cs374





MST - minimum total weight spanning tree

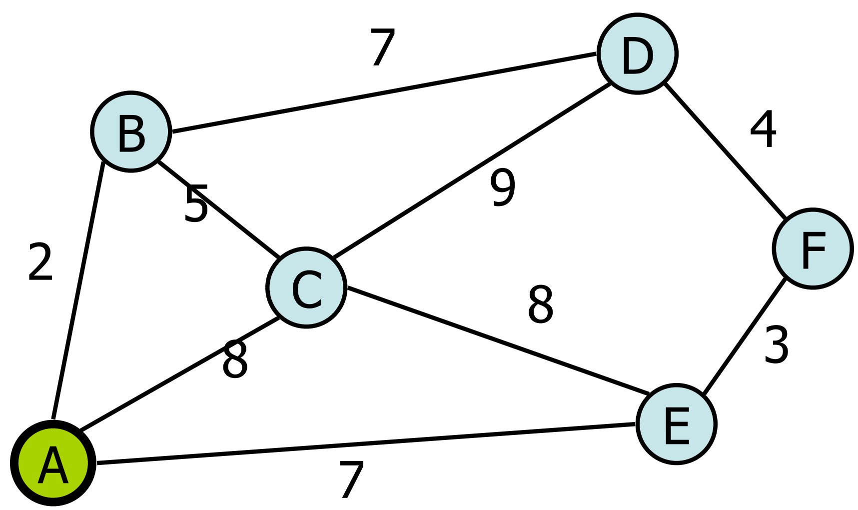
Theorem suggests an algorithm...



Example of Prim’s algorithm -

Initialize structure:

1. For all v, d[v] = “infinity”, p[v] = null
2. Initialize source: d[s] = 0
3. Initialize priority (min) queue
4. Initialize set of labeled vertices to ∅.



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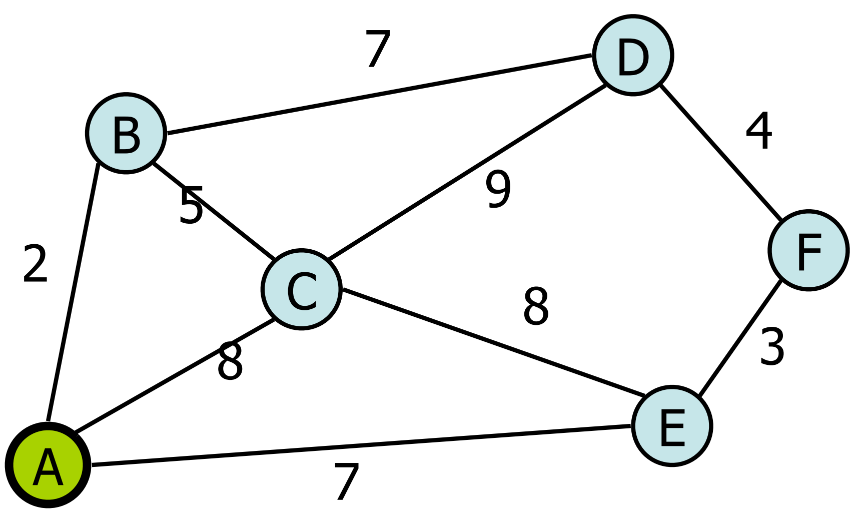

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4. Initialize set of labeled vertices to ∅.

Repeat these steps n times:

- Find & remove minimum d[] unlabelled vertex: v
- Label vertex v
- For all unlabelled neighbors w of v,

  If cost(v,w) < d[w]

  d[w] = cost(v,w)

  p[w] = v

Prim's Algorithm (undirected graph with unconstrained edge weights):

Initialize structure:

1. For all v, d[v] = "infinity", p[v] = null
2. Initialize source: d[s] = 0
3. Initialize priority (min) queue
4. Initialize set of labeled vertices to ∅.

Repeat these steps n times:

- Remove minimum d[] unlabeled vertex: v
- Label vertex v (set a flag)
- For all unlabeled neighbors w of v,

  If cost(v,w) < d[w]

  d[w] = cost(v,w)

  p[w] = v

	adj mtx	adj list
heap	$O(n^2 + m \log n)$	$O(n \log n + m \log n)$
Unsorted array	$O(n^2)$	$O(n^2)$

Which is best?

Depends on density of the graph:

Sparse

Dense